Present Value of an Annuity

Finite Math

24 February 2017

Present Value - Set Up

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We will look at making a large deposit in order to have a fund which we can make constant withdraws from. We make an initial deposit, then make withdraws at the end of each interest period. We should have a balance of $0 at the end of the predetermined amount of time the fund should last.

Example

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How much should you deposit into an account paying 6% *compounded semiannually in order to be able to withdraw* $\$2000$ *every* 6 *months for* 2 *years? (At the end of the* 2 *years, there should be a balance of* $\$0$ *in the account.)*

Solution

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So adding up the present values of all these will give us the amount of money we should deposit into the account now

$$D = \$2000(1.03)^{-1} + \$2000(1.03)^{-2} + \$2000(1.03)^{-3} + \$2000(1.03)^{-4} = \$7434.20$$

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Definition (Present Value of an Ordinary Annuity)

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Note that the payments are made at the end of each period.

Now You Try It!

Example

How much should you deposit in an account paying 8% *compounded quarterly in order to receive quarterly payments of* $\$1,000$ *for the next* 4 *years?*

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Solution

$\$13,577.71$

Combination Example

An interesting application of this in conjunction with sinking funds is saving for retirement.

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The full retirement age in the US is 67 *for people born in* 1960 *or later. Suppose you start saving for retirement at* 27 *years old and you would like to save enough to withdraw* $\$40,000$ *per year for the next* 20 *years. If you find a retirement savings account (for example, a Roth IRA) which pays* 4% *interest compounded annually, how much will you have to deposit per year from age* 27 *until you retire in order to be able to make your desired withdraws?*